

Emergent Universe in Chameleon, $f(R)$ and $f(T)$ Gravity Theories

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In this work, we have considered emergent universe in generalized gravity theories like Chameleon, $f(R)$ and $f(T)$ gravities. We have reconstructed the potential of Chameleon field under emergent scenario of the universe and observed its increasing nature with evolution of the universe. We have revealed that in the emergent universe scenario, the equation of state parameter behaves like quintessence in the case of $f(R)$ gravity and like phantom in the case of $f(T)$ gravity.

PACS numbers:

I. INTRODUCTION

The central paradigm for structure formation in the universe is the inflationary scenario. Under very general conditions, inflation is future eternal, in the sense that once inflation has started, most of the volume of the universe will remain in an inflating state [1]. We consider the total density $\Omega(t) = 1 + \frac{k}{a^2 H^2}$, where, k , a and H denote the curvature, scale factor and Hubble parameter respectively. Inflation drives the curvature term towards 0, which does not imply that $k = 0$. However, the standard inflationary model is based on a flat ($k = 0 \iff \Omega_0 = 1$) Friedmann-Robertson-Walker (FRW) geometry, motivated by the fact that inflationary expansion rapidly wipes out any original spatial curvature. Leading alternatives to inflation that are motivated by recent advances in string/M-theory are the pre-big bang [2] and ekpyrotic/cyclic [3] scenarios, respectively. The spatial curvature of the real universe is in principle determined by observations. The recent WMAP data seems to point to a universe that is close to (but not quite) flat, with a total density parameter of $\Omega_{tot} = 1.02 \pm 0.02$. Theory will have to give way to data if the data clearly tell us that $\Omega_0 > 1$ ([4],[5]). If Ω_0 is taken as 1.02, then the power spectra of CMB anisotropies and matter can show testable differences from the standard flat model [6].

The singularity theorems assume that either the universe has open space sections, implying $k = 0$ or -1 ; or the Hubble expansion rate $H = \frac{\dot{a}}{a}$ is bounded away from zero in the past. There are inflationary universes that evade these constraints and hence avoid the conclusions of the theorems [7]. Ellis and Maartens [4] considered closed models in which $k = +1$ and H can become zero, so that both of the assumptions of the inflationary singularity theorems are violated. In this paper [4], it was shown that if $k = +1$ then there are closed inflationary models that do not bounce, but inflate from a static beginning, and then reheat in the usual way.

The inflationary universe emerges from a small static state that has within it the seeds for the development of the microscopic universe and it is called Emergent Universe scenario. The universe has a finite initial size, with a finite amount of inflation occurring over an infinite time in the past and with inflation then coming to an end via reheating in the standard way. Reference [8] (see also [11]) summarized the features of emergent universe as:

1. the universe is almost static at the finite past ($t \rightarrow -\infty$) and isotropic, homogeneous at large scales;
2. it is ever existing and there is no timelike singularity;
3. the universe is always large enough so that the classical description of space-time is adequate;
4. the universe may contain exotic matter so that the energy conditions may be violated;

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5. the universe is accelerating as suggested by recent measurements of distances of high redshift type Ia supernovae.

Ellis et al [9] provided a realization of a singularity-free inflationary universe in the form of a simple cosmological model dominated at early times by a single minimally coupled scalar field with a physically based potential. Mukherjee et al [8] presented a general framework for an emergent universe scenario and showed that emergent universe scenarios are not isolated solutions and they may occur for different combinations of radiation and matter. Campo et al [10] studied the emergent universe model in the context of a self-interacting Jordan-Brans-Dicke theory and showed that the model presents a stable past eternal static solution which eventually enters a phase where the stability of this solution is broken leading to an inflationary period. Debnath [11] discussed the behaviour of different stages of the evolution of the emergent universe considering that the universe is filled with normal matter and a phantom field. Mukherji and Chakraborty [12] developed Einstein-Gauss-Bonnet (EGB) theory in the emergent universe scenario. They have considered the Friedman-Lemaître-Robertson-Walker cosmological model in Horava gravity and the emergent scenario for all values of the spatial curvature. Paul et al [13] predicted the range of the permissible values for the parameters associated with the constraints on exotic matter needed for an emergent universe. The emergent universe in Horava gravity was studied by Mukherji and Chakraborty [14].

Many theoretical models that anyhow describing the accelerated expansion of the universe and which appears to fit all currently available observations are affected by significant fine-tuning problems related to the vacuum energy scale and therefore it is important to investigate alternatives to this description of the Universe [15]. There exist several other approaches to the theoretical description of the accelerated expansion of the universe. One of these is a modified gravity theories. Studies of the physics of these theories is however hampered by the complexity of the field equations, making it difficult to obtain both exact and numerical solutions which can be compared with observations. These problems can be reduced somewhat by using the theory of dynamical systems, which provides a relatively simple method for obtaining exact solutions and a description of the global dynamics. Modified gravity constitutes an interesting dynamical alternative to Λ CDM cosmology in that it is also able to describe with success the current acceleration in the expansion of our universe, the present dark energy epoch [16]. Presumably the simplest modified gravity model of dark energy is so-called $f(R)$ gravity in which f is a function in terms of a Ricci scalar R ([17], [18], [19], [20], [21], [22]). The $f(R)$ theories do not seem to introduce any new type of matter and can lead to late time acceleration [23]. Another modified gravity theory is the Chameleon field theory where the scalar field interacts with some kind of matter, behaving as perfect fluid. The Lagrangian of the model contains a term, where the matter interacts with the effective metric (physical metric, multiplied by a conformal factor depending on the scalar field)[24]. Brax et al [23] reviewed both $f(R)$ gravity and Chameleon field theories and discussed association between them. Another interesting sort of modified theories is so-called $f(T)$ gravity (where T is the torsion) ([25], [26], [27]). Recently, it is shown that such $f(T)$ gravity theories also admit the accelerated expansion of the universe without resorting to dark energy ([27], [28]). In the present paper, we have discussed the emergent universe scenario in the generalized gravity theories like Chameleon gravity, $f(R)$ gravity and $f(T)$ gravity.

The organization of the paper is as follows: In section II, we have given the basic field equations in the Einstein's gravity. We have chosen some particular form of scale factor for emergent scenario of the universe and made some restrictions upon the parameters. In section III, we have discussed the Chameleon field gravity model and shown the behaviour of the potential function for emergent scenario. In section IV and V, we have studied the $f(R)$ and $f(T)$ gravity models respectively and shown the nature of equation of states in emergent scenarios of the universe. Finally, some conclusions have been drawn in section VI.

II. BASIC EQUATIONS

For a FRW spacetime, the line element is given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

where $a(t)$ is the scale factor and k ($= 0, \pm 1$) is the curvature scalar. Now consider the Hubble parameter (H) and the deceleration parameter (q) in terms of scale factor as

$$H = \frac{\dot{a}}{a} , \quad q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}$$

So the Einstein field equations are given by (choosing $c = 1$)

$$H^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3} \quad (2)$$

and

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p) \quad (3)$$

and the energy conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (4)$$

where ρ and p are respectively the density and pressure of the fluid.

For emergent universe, the scale factor is chosen as [8]

$$a = a_0 (\beta + e^{\alpha t})^n \quad (5)$$

where the constant parameters are restricted as follows [14]:

1. $a_0 > 0$ for the scale factor a to be positive,
2. $\beta > 0$, to avoid any singularity at finite time (big-rip),
3. $\alpha > 0$ or $n > 0$ for expanding model of the universe,
4. $\alpha < 0$ and $n < 0$ implies big bang singularity at $t = -\infty$.

For this choice of the scale factor, the Hubble parameter, its derivatives are given by

$$H = \frac{n\alpha e^{\alpha t}}{e^{\alpha t} + \beta} , \quad \dot{H} = \frac{n\beta\alpha^2 e^{\alpha t}}{(e^{\alpha t} + \beta)^2} , \quad \ddot{H} = \frac{n\beta\alpha^3 e^{\alpha t}(\beta - e^{\alpha t})}{(e^{\alpha t} + \beta)^3}$$

Here, H , \dot{H} , and \ddot{H} tend to 0 as $t \rightarrow -\infty$. We have seen that $\dot{H} > 0$ for emergent scenario. From (2) and (3), it can be observed that the emergent scenario is possible for (i) flat ($k = 0$) universe in phantom stage only, (ii) open ($k = -1$) universe only for phantom stage if $H > \frac{1}{a}$ and (iii) closed ($k = +1$) universe in quintessence stage if $\dot{H} < \frac{1}{a^2}$ and in phantom stage if $\dot{H} > \frac{1}{a^2}$.

Also we calculated the cosmographic parameters [29, 30]

$$\begin{aligned} q &= -\frac{1}{a} \frac{d^2 a}{dt^2} H^{-2} \\ j &= \frac{1}{a} \frac{d^3 a}{dt^3} H^{-3} \\ s &= \frac{1}{a} \frac{d^4 a}{dt^4} H^{-4} \\ l &= \frac{1}{a} \frac{d^5 a}{dt^5} H^{-5} \end{aligned} \quad (6)$$

which are usually referred to as the deceleration, jerk, snap, and lerk parameters. After calculating the cosmographic parameters for the emergent universe we have plotted them against cosmic time t in the figures 1a and 1b. It is observed in figure 1a that the deceleration parameter is staying at negative level. This indicates the ever accelerating nature of the emergent universe. In figure 1b we observe that the snap, jerk and lark parameters are increasing function of the cosmic time in the emergent universe.

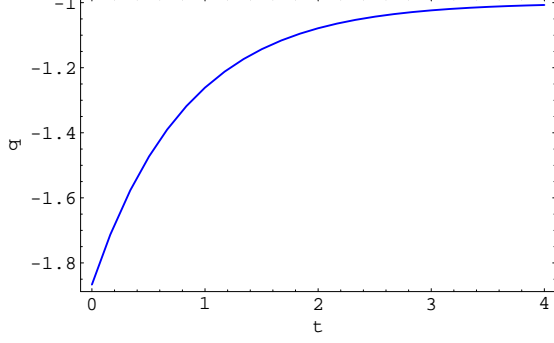


Fig.1a

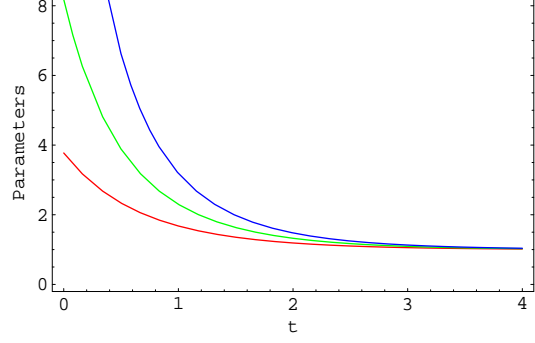


Fig.1b

Fig.1a shows the deceleration parameter q for emergent universe. Fig.1b shows snap parameter s (green line, middle), jerk parameter j (red line, bottom) and lerk parameter l (blue line, top) for emergent universe.

III. EMERGENT SCENARIO IN CHAMELEON GRAVITY

In the flat homogeneous universe, we consider that the relevant action given by [31, 32, 33]

$$S = \int \sqrt{-g} d^4x \left[f(\phi) \mathcal{L} + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + \frac{\mathcal{R}}{16\pi G} - V(\phi) \right] \quad (7)$$

where ϕ is the Chameleon scalar field and $V(\phi)$ is the Chameleon potential. Also, \mathcal{R} is the Ricci scalar and G is the Newtonian constant of gravity, $f(\phi) \mathcal{L}$ is the modified matter Lagrangian and $f(\phi)$ is an analytic function of ϕ . The variation of action (9) with respect to the metric tensor components in a FRW cosmology yields the field equations (choosing $8\pi G = 1$),

$$H^2 = \frac{1}{3} \left[\rho f + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] - \frac{k}{a^2} \quad (8)$$

and

$$\dot{H} = \frac{1}{2} \left[-p f - \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] + \frac{k}{a^2} \quad (9)$$

The conservation equation and the wave equation in presence of Chameleon field are given by [31]

$$\frac{\partial}{\partial t}(\rho f) + 3H(p + \rho)f = (p + \rho)\dot{f} \quad (10)$$

and

$$3H\dot{\phi}^2 + \dot{\phi}\ddot{\phi} + \dot{V} + (p + \rho)\dot{f} = 0 \quad (11)$$

The early universe contains a standard scalar field ϕ with energy density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and pressure $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ and possibly also ordinary matter with energy density ρ and pressure $p = w\rho$, where $-\frac{1}{3} \leq w \leq 1$. The cosmological constant is absorbed into the potential V . There are no interactions between matter and the scalar field. Following references [31,32] we make the simplifying assumption

$$V = V_0 \dot{\phi}^2 \quad (12)$$

where, V_0 is a positive constant. Using this simplifying assumption in the Chameleon field equation along with the specified form of scale factor (5) and equations (8) and (9), we get (after simplification) the form of ϕ in the emergent scenario as

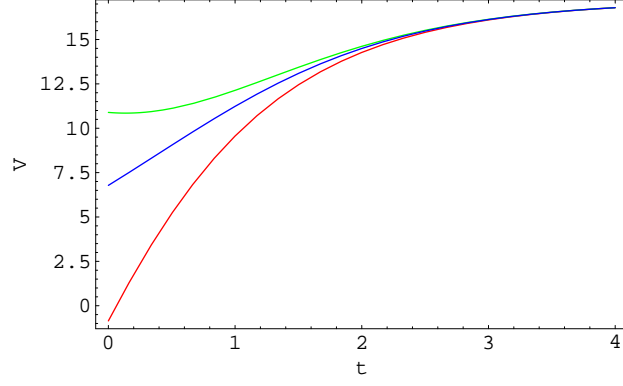


Fig.2a. This figure shows the variation of potential V of the Chameleon field with cosmic time t in the emergent universe scenario. The cases of flat ($k = 0$) (the blue line, middle), open ($k = -1$) (the red line, bottom) and closed ($k = 1$) (the green line, top) universe are considered.

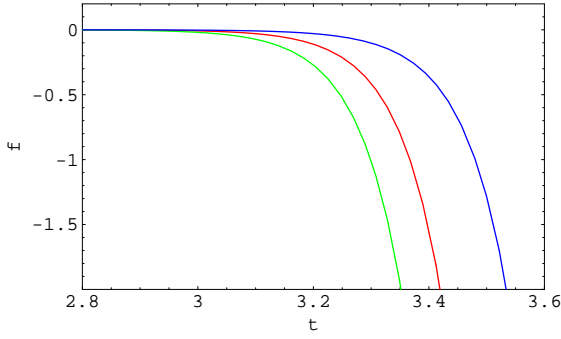


Fig.2b

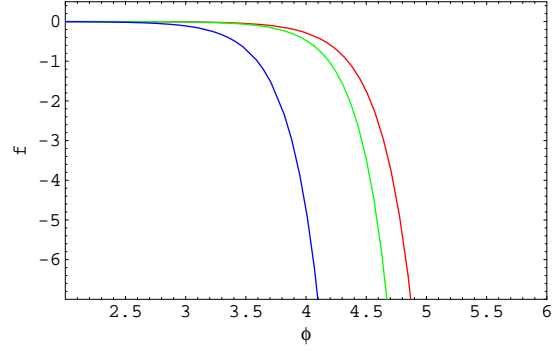


Fig.2c

Fig.2b shows the variation of the analytic function f of the Chameleon field with cosmic time t in the emergent universe scenario. In this figure, the cases of flat ($k = 0$) (the blue line, top), open ($k = -1$) (the red line, middle) and closed ($k = 1$) (the green line, bottom) universe are considered. Fig.2c shows the variation of the analytic function f of the Chameleon field with ϕ in the emergent universe scenario. In this figure, the cases of flat ($k = 0$) (the blue line, bottom), open ($k = -1$) (the red line, top) and closed ($k = 1$) (the green line, middle) universes are considered.

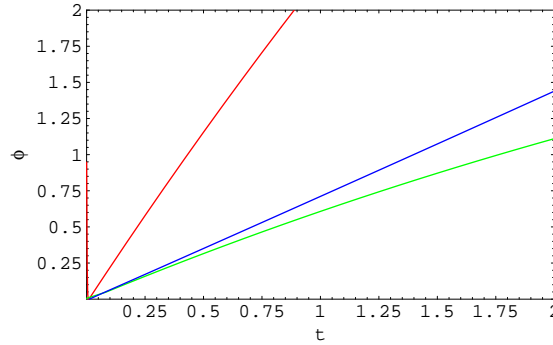


Fig.3

Fig. 3 represents the evolution of the scalar field ϕ of the Chameleon field with time t in the emergent universe scenario for open ($k = -1$) (the red line, top), closed ($k = 1$) (the green line, bottom) and flat ($k = 0$) (the blue line, middle), universe respectively.

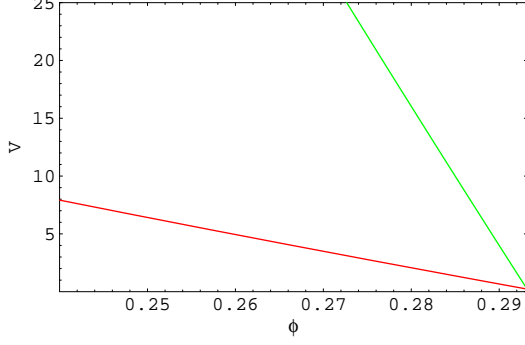


Fig.4a

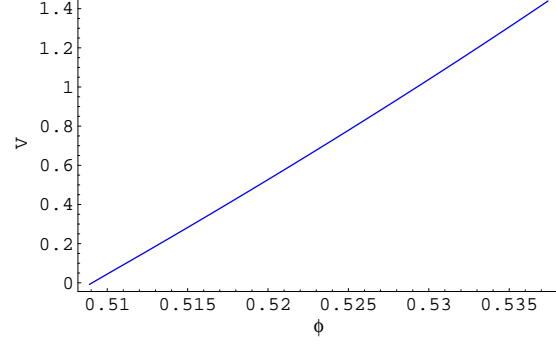


Fig.4b

Fig. 4a shows the potential V against the scalar field ϕ in the cases of ($k = -1$) (the red line, bottom) and closed ($k = 1$) (the green line, top). Fig. 4b shows the potential V against the scalar field ϕ in the case of flat $k = 0$ universe.

$$\phi = \int \sqrt{\frac{2(k(1+3kw)(e^{\alpha t} + \beta)^{-2n} + e^{\alpha t} n \alpha^2 (e^{\alpha t} + \beta)^{-2} (3e^{\alpha t} n(1+w) + 2\beta) a_0^2)}{a_0^2 (-1+w+2(1+w)V_0)}} dt \quad (13)$$

Using (5), (8), (12) and (13) and the $\rho = \rho_0 a^{-3(1+w)} f^w$ (from (10)) for Chameleon field from reference [31], we express $f(\phi)$ as a function of t as follows

$$f = \left[\frac{((e^{t\alpha} + \beta)^n a_0)^{1+3w} (k(e^{t\alpha} + \beta)^2 (-4 + 3(1-k)w + (4 + 6(1-k)w)V_0) - 2e^{t\alpha} n \alpha^2 (e^{t\alpha} + \beta)^{2n} a_0^2 (3e^{t\alpha} n + \beta(1 + 2V_0)))}{((e^{t\alpha} + \beta)^2 (-1 + w + 2(1+w)V_0) \rho_0)} \right]^{\frac{1}{1+w}} \quad (14)$$

We see that $(e^{\alpha t} + \beta)^{-2} (3e^{\alpha t} n(1+w) + 2\beta) a_0^2$ is always positive. Thus from (13) we see that emergent scenario is possible for flat, open and closed universe under the following conditions:

- In the case of $k = 0$, $\dot{\phi}^2 > 0$ for $V_0 > \frac{1-w}{2(1+w)}$
- In the case of $k = -1$, $\dot{\phi}^2 > 0$ for $w > \frac{1-e^{\alpha t} n \alpha^2 (e^{\alpha t} + \beta)^{2(-1+n)} (3ne^{\alpha t} + 2\beta) a_0^2}{3(1+e^{2\alpha t} n^2 \alpha^2 a_0^2 (e^{\alpha t} + \beta)^{2(-1+n)})}$ and $V_0 > \frac{1-w}{2(1+w)}$
- In the case of $k = 1$, $\dot{\phi}^2 > 0$ for $w > -\frac{1+e^{\alpha t} n \alpha^2 (e^{\alpha t} + \beta)^{2(-1+n)} (3ne^{\alpha t} + 2\beta) a_0^2}{3(1+e^{2\alpha t} n^2 \alpha^2 a_0^2 (e^{\alpha t} + \beta)^{2(-1+n)})}$ and $V_0 > \frac{1-w}{2(1+w)}$.

Thus, under the following conditions we consider the cases of flat ($k = 0$), open ($k = -1$) and closed ($k = +1$) universe. In figure 2a, we plot the potential against cosmic time. We find that the potential is increasing with time in all of the three cases. While plotting, the constant parameters are chosen according to the above four constraints. Evolution of $f(\phi)$ is presented in figures 2b and 2c. In all of the three situations $f(\phi)$ is staying at negative level and is showing a falling behaviour with cosmic time as well as the scalar field ϕ . For open ($k = -1$), closed ($k = +1$), and flat ($k = 0$) universe, the evolution of the scalar field ϕ exhibits an increasing pattern with evolution of the universe. These are presented in figure 3 where the scalar field ϕ is plotted against cosmic time t . Figure 4a shows that potential V is decreasing with the scalar field ϕ for $k = -1$ and $k = +1$. However, figure 4b shows that for $k = 0$ i.e. for open universe, V is increasing with ϕ .

IV. EMERGENT SCENARIO IN $f(R)$ GRAVITY

Recently, motivated by astrophysical data which indicate that the expansion of the universe is accelerating, the modified theory of gravity (or $f(R)$ gravity) which can explain the present acceleration without introducing

dark energy, has received intense attention [34-40]. In a recent work, Nojiri and Odinstov [38] suggested two realistic $f(R)$ and one $F(G)$ modified gravities characterized by the presence of the effective cosmological constant epochs in such a way that early-time inflation and late-time cosmic acceleration are naturally unified within single model. In another work, Nojiri and Odintsov [39] proposed another class of modified $f(R)$ gravity which unifies R^m inflation with Λ CDM era. Cognola et al [40] demonstrated a class of viable modified $f(R)$ gravities describing inflation and the onset of accelerated expansion. In $f(R)$ gravity action is described by an arbitrary function of the scalar curvature R . Extensive review of $f(R)$ gravity is available in Nojiri and Odintsov [21, 22]. The action of $f(R)$ gravity is given by [44-47]

$$S = \int d^4x \sqrt{-g} \left[\frac{f(R)}{2\kappa^2} + \mathcal{L}_{matter} \right] \quad (15)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, \mathcal{L}_{matter} is the matter Lagrangian and $\kappa^2 = 8\pi G$. The $f(R)$ is a non-linear function of the Ricci curvature R that incorporates corrections to the Einstein-Hilbert action which is instead described by a linear function $f(R)$. The gravitational field equations in this theory are

$$H^2 = \frac{\kappa^2}{3f'(R)}(\rho + \rho_c) \quad (16)$$

$$\dot{H} = -\frac{\kappa^2}{2f'(R)}(\rho + p + \rho_c + p_c) \quad (17)$$

where ρ_c and p_c can be regarded as the energy density and pressure generated due to the difference of $f(R)$ gravity from general relativity given by [45]

$$\rho_c = \frac{1}{\kappa^2} \left[\frac{1}{2}(-f(R) + Rf'(R)) - 3H\dot{R}f''(R) \right] \quad (18)$$

$$p_c = \frac{1}{\kappa^2} \left[\frac{1}{2}(f(R) - Rf'(R)) + (2H\dot{R} + \ddot{R})f''(R) + \dot{R}^2 f'''(R) \right] \quad (19)$$

where, the scalar tensor $R = 6(\dot{H} + 2H^2)$ [39,45]. Here, $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Here, ρ and p are the matter energy density and pressure respectively. In the spatially flat universe, i.e. $k = 0$, with the matter as dust, namely $p = 0$, following Feng [45], $p = 3H_0^2 \Omega_{m0} e^{-3x}$ with $x = \ln a$, $\Omega_{m0} = \rho_{m0}/3H_0^2$ and H_0 is the present Hubble parameter. Following references [36] we choose $f(R)$ as

$$f(R) = R + \xi R^\mu + \zeta R^{-\nu} \quad (20)$$

where, constants $\mu > 0$; $\nu > 0$. Subsequently, we get from equation (17)

$$\begin{aligned} \dot{H} = & (a^3 H R (\zeta \nu (1 + \nu) + R^{\mu+\nu} (-1 + \mu) \mu \xi) \dot{R} + a^3 (\zeta \nu (1 + \nu) (2 + \nu) - R^{\mu+\nu} (-2 + \mu) (-1 + \mu) \mu \xi) \dot{R}^2 + \\ & a^3 R (-\zeta \nu (1 + \nu) - R^{\mu+\nu} (-1 + \mu) \mu \xi) \ddot{R} - 3R^{3+\nu} \kappa^2 H_0^2 \Omega_{m0}) \times \\ & (2a^3 R^2 (-\zeta \nu + R^\nu (R + R^\mu \mu \xi)))^{-1} \end{aligned} \quad (21)$$

As both R and H are functions of the scale factor a and its derivatives, it seems that the equation (21) is set for yielding the solution for the scale factor. However, it involves fourth-order derivatives of a (R already contains \ddot{a}) and is highly nonlinear. This makes it difficult to obtain a completely analytic solution for a . Mukherjee et al [8] obtained the general solution of the scale factor for emergent universe in the form presented in equation (6) without referring to the actual source of the energy density. So we use equation (6) as the choice of scale factor for emergent nature of the universe and reconstruct ρ_c and p_c based on R in the emergent universe and are expressed as follows

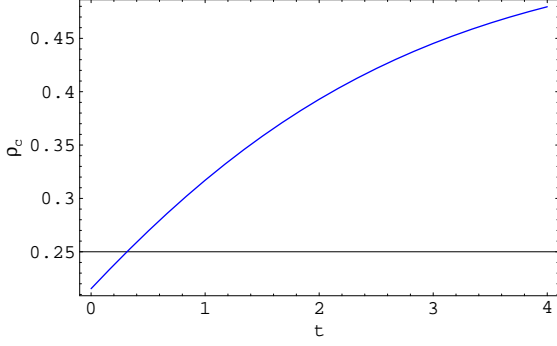


Fig.5a

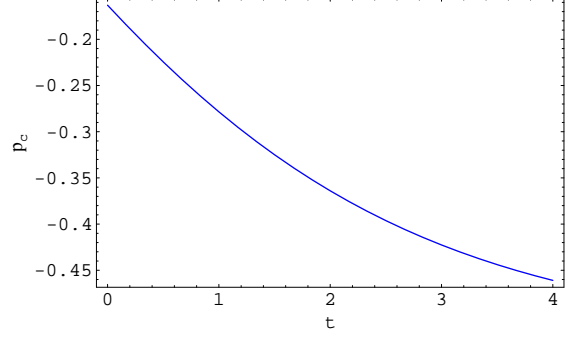


Fig.5b

Figs. 5a and 5b represent the evolution of the energy density ρ_c and pressure p_c generated due to the difference of $f(R)$ gravity from general relativity.

$$\rho_c = \frac{1}{(2ne^{t\alpha} + \beta)^2 \kappa^2} (2^{-1-\nu} 3^{-\nu} \varrho^{-\nu} ((1+\nu)(-4e^{2t\alpha} n^2 - e^{t\alpha}(-\nu + 4n(1+\nu))\beta - (1+\nu)\beta^2)\zeta + \varrho^{\mu+\nu}(2^{2+\mu+\nu} 3^{\mu+\nu} e^{2t\alpha} n^2(-1+\mu) - 6^{\mu+\nu} e^{t\alpha} \times (4n(-1+\mu) - \mu)(-1+\mu)\beta + (2^{1+\mu+\nu} 3^{\mu+\nu} \mu - 6^{\mu+\nu}(1+\mu^2))\beta^2)\xi)) \quad (22)$$

$$p_c = \frac{1}{6\kappa^2} (\varrho^{-\nu} (2^{-\nu} 3^{1-\nu} ((1+\nu)\zeta - 6^{\mu+\nu}(-1+\mu)\xi \varrho^{\mu+\nu})) + \frac{1}{n\alpha(2ne^{t\alpha} + \beta)^2} (6^{-\nu} e^{-t\alpha} \beta (e^{t\alpha}(-1+4n) + \beta)(e^{t\alpha}(1+2n\alpha) + \beta) \times + (\nu(1+\nu)\zeta + 6^{\mu+\nu}(-1+\mu)\mu\xi \varrho^{\mu+\nu})) + \frac{1}{n(2ne^{t\alpha} + \beta)^3} (6^{-\nu} e^{-t\alpha} \beta^2 (e^{t\alpha}(-1+4n) + \beta)^2 (-\nu(1+\nu)(2+\nu)\zeta + + 6^{\mu+\nu}(-2+\mu)(-1+\mu)\mu\xi \zeta^{\mu+\nu} \quad (23)$$

where

$$\varrho = \frac{n\alpha^2 e^{t\alpha} (2ne^{t\alpha} + \beta)}{(e^{t\alpha} + \beta)^2} \quad (24)$$

The ρ_c and p_c are plotted in figures 5a and 5b respectively. The figures show that although ρ_c is increasing with cosmic time t , the negative p_c is falling with cosmic time. While summarizing the attractiveness of the modified gravity approach, Nojiri and Odintsov [35] stated that "modified gravity may naturally describe the transition from non-phantom phase to phantom one without necessity to introduce the exotic matter". Abdalla et al [49] had shown in an earlier study that the modified gravity minimally coupled with the usual (or quintessence) matter may reproduce the quintessence (or phantom) evolution phase for the dark energy universe in an easier way than without such coupling. In a more recent study, Bamba et al [50] studied a viable model of modified gravity in which the transition from the de Sitter universe to the phantom phase can occur. Motivated by these earlier studies we examine the behaviour of the equation of state parameter in $f(R)$ gravity. The equation of state parameter plotted in figure 6 is found to stay above -1 . This indicates quintessence behaviour. This means that evolution from quintessence from phantom is not available in the case of $f(R)$ gravity in emergent universe. From figure 7 we find the increasing behavior of $f(R)$ with the evolution of the universe.

V. EMERGENT SCENARIO IN $f(T)$ GRAVITY

Recently, models based on modified teleparallel gravity were presented as an alternative to inflationary models. The theory so obtained is called as the $f(T)$ theory [51-57]. In this theory instead the curvature defined via

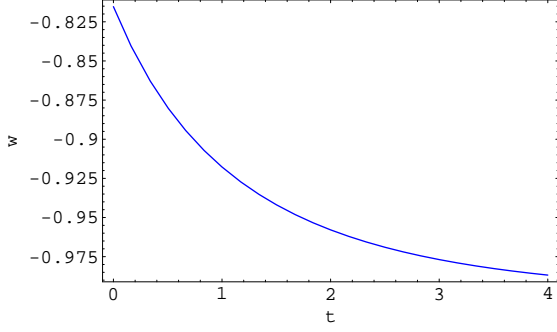


Fig.6

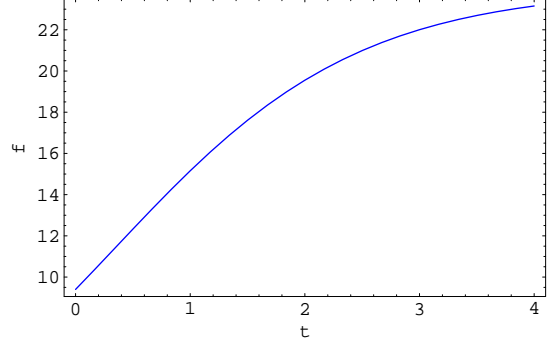


Fig.7

Fig.6 shows the evolution of the equation of state $w_c = \frac{p_c}{\rho_c}$ with cosmic time t .

Fig.7 represents the evolution of $f(R)$ with cosmic time t .

the Levi-Civita connection, the so-called Weitzenbock connection is used. But in this case the theory has no curvature but instead torsion. Similar to general relativity where the action is the curvature scalar, the action of teleparallel gravity is a torsion scalar T ([51], [52], [58]). The action of $f(T)$ theory is obtained by replacing T in the action of teleparallel gravity by $T + f(T)$ [53]. We start with the following action for the $f(T)$ gravity [52]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [T + f(T) + \mathcal{L}_m] \quad (25)$$

where T is the torsion scalar, $f(T)$ is general differentiable function of the torsion and \mathcal{L}_m corresponds to the matter Lagrangian and $\kappa^2 = 8\pi G$. The torsion scalar T is defined as

$$T = S_\rho^{\mu\nu} T_{\mu\nu}^\rho \quad (26)$$

with

$$S_\rho^{\mu\nu} = \frac{1}{2} (K_\rho^{\mu\nu} + \delta_\rho^\mu T_\theta^{\theta\nu} - \delta_\rho^\nu T_\theta^{\theta\mu}) \quad (27)$$

where, $K_\rho^{\mu\nu}$ is the contorsion tensor, which equals the difference between Weitzenbock and Levi-Civita connections. So $f(T)$ gravity uses the curvatureless Weitzenbock connection.

The modified Friedmann equations of motion are ([27], [56])

$$H^2 = \frac{8\pi G}{3} \rho_m - \frac{f(T)}{6} - 2f'(T)H^2 \quad (28)$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f'(T) - 12H^2 f''(T)} \quad (29)$$

where, ρ_m and p_m respectively for the energy density and pressure of the matter content of the universe, with equation-of-state parameter $w_m = p_m/\rho_m$. Since we are interested in the late time universe, we consider only non-relativistic matter (cold dark matter and baryon), we have $p_m = 0$. By comparing the above modified Friedmann equations with the ordinary ones in general relativity one can obtain the energy densities and pressure for $f(T)$ gravity as [26]

$$\rho_T = \frac{1}{2\kappa^2} (2T f'(T) - f(T) + 6H^2) \quad (30)$$

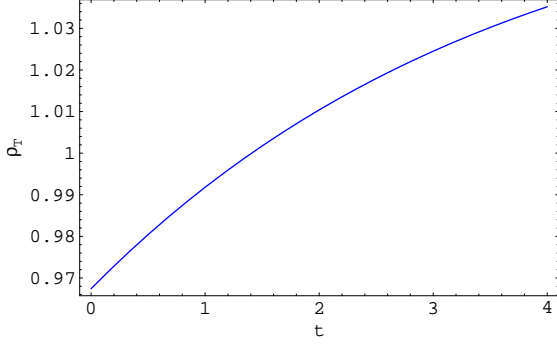


Fig.8a

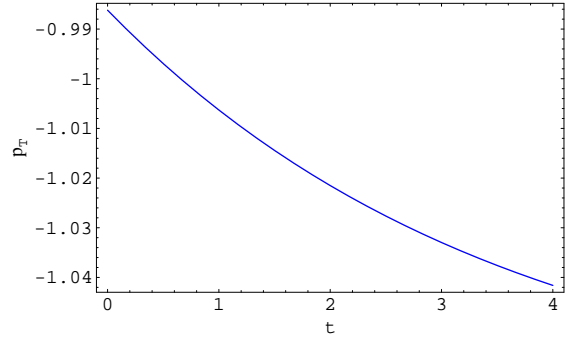


Fig.8b

Figs. 8a and 8b represent the evolution of the energy density ρ_T and pressure p_T respectively against cosmic time t .

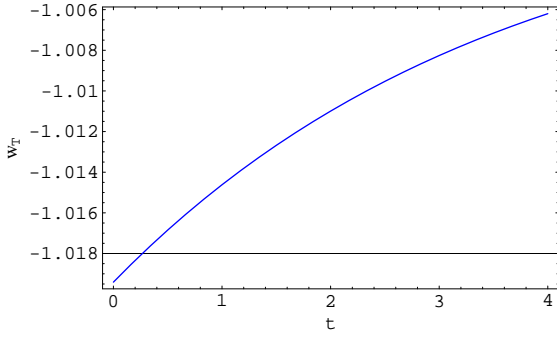


Fig.9

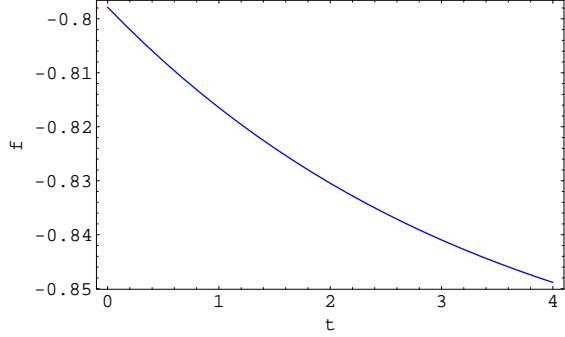


Fig.10

Fig. 9 shows the evolution of the equation of state $w_T = \frac{p_T}{\rho_T}$ with cosmic time t .

Figs. 10 represents the evolution of $f(T)$ with cosmic time t .

and

$$p_T = -\frac{1}{2\kappa^2} \left[-8\dot{H}T f''(T) + (2T - 4\dot{H})f'(T) - f(T) + 4\dot{H} + 6H^2 \right] \quad (31)$$

and the equation of state

$$w_T = -1 - \frac{8\dot{H}T f''(T) + 4\dot{H}f'(T)}{f(T) - 2Tf'(T)} \quad (32)$$

As we are considering the emergent universe scenario, we use the scale factor a as in equation (6). Subsequently, H gets the form in (7). Following reference [49] we take $f(T)$ as

$$f(T) = \gamma T + \lambda T^m \quad (33)$$

where, $m > 0$ and $T = -6H^2$. Consequently,

$$\dot{H} = -\frac{36H^2\kappa^2 H_0^2 \Omega_{m0}}{a^3(6H^2(1+\gamma) - (-H^2)^m(6^m + 2^{1+m}3^m(-1+m))m\lambda)} \quad (34)$$

The above equation is non-linear in H and hence it is difficult to get an analytical solution of a . Thus, like the $f(R)$ gravity we take the scale factor in the form (6) to consider the emergent universe scenario. Using the above forms of a and $f(T)$ we reconstruct ρ_T , p_T and the equation of state parameter w_T . Forms of ρ_T and p_T are obtained as follows

$$\rho_T = \frac{\kappa^2}{2} [-6^m \eta^m \lambda + \frac{1}{(e^{t\alpha} + \beta)^5} (6e^{2t\alpha} n^2 \alpha^2 (e^{3t\alpha} + 3e^{2t\alpha} \beta + 3e^{t\alpha} \beta^2 + \beta^2 + e^{3t\alpha} \gamma + 3e^{2t\alpha} \beta \gamma + 24e^{2t\alpha} n^2 \alpha^3 \beta \gamma) + (3e^{t\alpha} + \beta) \beta^2 \gamma - 2^{2+m} 3^m m \alpha \beta (e^{t\alpha} + \beta)^2 \lambda \eta)] \quad (35)$$

$$p_T = -\frac{\kappa^2}{2} [\frac{2e^{t\alpha} n \alpha^2 (ne^{t\alpha} (3+2\gamma) + 2\beta)}{(e^{t\alpha} + \beta)^2} - 6\gamma \eta + \frac{1}{(e^{t\alpha} + \beta)^5} \times (8e^{t\alpha} n \alpha^3 \beta (3ne^{t\alpha} + \beta) (6e^{2t\alpha} n^2 \alpha^2 \gamma - m\eta (6^m e^{2t\alpha} + 2^{1+m} 3^m e^{t\alpha} \beta + 6^m \beta^2) \gamma)) + \frac{1}{(e^{t\alpha} + \beta)^8} (96e^{3t\alpha} n^3 \alpha^6 \beta^2 (6e^{2t\alpha} n^2 \alpha^2 (e^{t\alpha} - 2\beta) \gamma + m\eta (-6^m e^{3t\alpha} + 2^{1+m} 3^m e^{2t\alpha} (-1+m) \beta + 6^m e^{t\alpha} (-1+4m) \beta^2 + 2^{1+m} 3^m m \beta^3) \lambda)))] \quad (36)$$

where,

$$\eta = - \left[\frac{n^2 \alpha^2 e^{2t\alpha}}{(e^{t\alpha} + \beta)^2} \right]^m \quad (37)$$

Figures 8a and 8b show the variation of ρ_T and p_T with cosmic time t . Which show that in the emergent scenario under $f(T)$ gravity the energy density increases with time, while the negative pressure falls with increase in the cosmic time. The equation of state parameter $w_T < -1$ as plotted against cosmic time in figure 9. This indicates the phantom scenario. In figure 10 we find the evolution of $f(T)$ with cosmic time.

VI. CONCLUDING REMARKS

In this work, we have considered emergent universe in generalized gravity theories like Chameleon, $f(R)$ and $f(T)$ gravities. While considering the Chameleon field, we have considered flat, open and closed universes. We have reconstructed the potential of Chameleon field under emergent universe scenario and observed its increasing nature with evolution of the universe. Also, we observed the increasing behavior of the associated scalar field in all of the flat, open and closed universes. We have derived the conditions for emergent scenario in the situations of flat, open and closed universes. Next, we have considered the emergent universe scenario under $f(R)$ gravity. Taking a particular form of $f(R)$ we have considered the energy density, pressure and subsequently the equation of state parameter. It has been revealed that the equation of state parameter exhibits quintessence like behaviour (above -1). However, considering the behaviour of the equation of state parameter under $f(T)$ gravity in the emergent universe scenario, we have seen it to lie below -1 . This indicates phantom like behaviour.

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